

# Compilers

3<sup>er</sup> course  
Spring Term

Alfonso Ortega: [alfonso.ortega@uam.es](mailto:alfonso.ortega@uam.es)  
Enrique Alfonseca: [enrique.alfonseca@uam.es](mailto:enrique.alfonseca@uam.es)



## Chapter 4: Syntactic Analyser

Part 4d2: *First and Next relationships*



## Symbols that can generate the empty word

### Procedure

- We say that a non-terminal symbol  $X$  can generate the empty word, and we write it

$$X \rightarrow^* \lambda$$

if:

- There is a production rule of the form  $X \rightarrow \lambda$
- There is a non-terminal symbol  $B$  such that
  - $B \rightarrow^* \lambda$ , and
  - There is a production rule of the form  $X \rightarrow B$ ,

3

## FIRST relationship

### Definition

- Formally,

Let  $G = \{\Sigma_N, \Sigma_T, P, A\}$  be a context-independent grammar.

If  $\alpha$  is a sentential form of the grammar, ( $\alpha \in (\Sigma_N \cup \Sigma_T)^*$ ), we shall call **first( $\alpha$ )** the set of terminal symbols that can be at the beginning of the strings derived from  $\alpha$ .

In the case that the empty string  $\lambda$  can also be derived from  $\alpha$ , then we shall say that  $\lambda$  also belongs to **first( $\alpha$ )**

4

## FIRST relationship

### Algorithm

- The following is an algorithm for calculating  $\text{first}(X) \quad \forall X \in \Sigma_N \cup \Sigma_T$ :

The following rules will be applied, until we cannot add any new non-terminal symbols, or  $\lambda$

1. Initialise  $\text{first}(X)$  as an array.
2. If  $X \in \Sigma_T$  then  $\text{first}(X) = \{X\}$ . Return it.
3. If  $X \rightarrow * \lambda \in P$  then insert  $\lambda$  in  $\text{first}(X)$ .
4. If  $X \rightarrow Y_1 Y_2 \dots Y_k \in P$  then
  - For  $i$  from 1 to  $k$ ,
    - Every element from  $\text{first}(Y_i)$  has to be in  $\text{first}(X)$
    - If  $Y_i \rightarrow * \lambda \in P$  then continue the loop;
    - Otherwise, break.

5

## FIRST relationship

### Algorithm

- Description of the algorithm to calculate  $\text{first}(\alpha) \quad \forall \alpha \in (\Sigma_N \cup \Sigma_T)^*$ :

If  $\alpha = X_1 X_2 \dots X_n$  then

- If  $X_i \rightarrow * \lambda \quad \forall i \in \{1, 2, \dots, n\}$  then  $\lambda \in \text{first}(\alpha)$ .
- If  $j \in \{1, 2, \dots, n\}$  is the first sub-index such that  $X_j \rightarrow * \lambda$  does not hold, then  $\text{first}(Y_j) \subseteq \text{first}(\alpha) \quad \forall i \in \{1, 2, \dots, j\}$

6

## NEXT relationship

### Definition

- Formally,

Let  $G = \{\Sigma_N, \Sigma_T, P, A\}$  be a context independent grammar.  
For any non-terminal symbol  $X$ , we shall call **next (X)** the set of terminal symbols that can be generated immediately after  $X$ .

If  $X$  can be generated at the rightmost position in sentential form generated by the grammar, then  $\$ \in \mathbf{next (X)}$

7

## NEXT relationship

### Algorithm

- The following pseudo-code calculates  $\mathbf{next (X)}$   $\forall X \in \Sigma_N$ :

1. Initialise  $\mathbf{next (X)}$  as an array.
2. If  $X=A$  (axiom) add  $\$$  to  $\mathbf{next (X)}$ .
3.  $\forall Y \rightarrow \alpha X \beta \in P, \beta \neq \lambda \wedge \lambda \notin \mathbf{first}(\beta) \Rightarrow \mathbf{first}(\beta) \subseteq \mathbf{next (X)}$ .
4.  $\forall Y \rightarrow \alpha X \beta \in P, \beta \neq \lambda \wedge \lambda \in \mathbf{first}(\beta)$ .
  - Then  $\mathbf{first}(\beta) - \{\lambda\} \subseteq \mathbf{next (X)}$  .
  - And  $\mathbf{next (Y)} \subseteq \mathbf{next (X)}$  .
5.  $\forall Y \rightarrow \alpha X \in P \Rightarrow \mathbf{next (Y)} \subseteq \mathbf{next (X)}$ .

8

## FIRST and NEXT sets

### Exercise

- Given the following grammar,

```
G=< {E,E',T,T',F},
    {+,*,(,),id}
    {
      E  → TE'
      E' → +TE' | λ
      T  → FT'
      T' → *FT' | λ
      F  → (E) | id
    },
E>
```

- Find all the symbols which can generate the empty word.
- Find the first set for all the non-terminal symbols.
- Find the next set for all the non-terminal symbols.

9

## FIRST and NEXT sets

### Exercise

- Given the following grammar,

```
G=< {E,A,B,C,D,E,F},
    {h,i,j,k,l}
    {
      E  → A | B
      A  → D | ED
      D  → CE | λ
      B  → BB | F | k | λ
      F  → hi | kkl
      C  → j | λ
    },
E>
```

- Find all the symbols which can generate the empty word.
- Find the first set for all the non-terminal symbols.
- Find the next set for all the non-terminal symbols.

10